**SGGS COLLEGIATE PUBLIC SCHOOL, SECTOR 26, CHANDIGARH**
**SUBJECT – MATHEMATICS**
**CLASS - XII**

Time: 3 hrs. M.M.: 80

**General Instructions:**

1. This question paper contains 38 questions.
2. This Question Paper is divided into 5 Sections A, B, C, D and E.
3. In Section A, Questions no. 1-18 are multiple choice questions (MCQs) and questions no. 19 and 20 are Assertion- Reason based questions of 1 mark each.
4. In Section B, Questions no. 21-25 are very short answer (VSA) type questions, carrying 02 marks. each.
5. In Section C, Questions no. 26-31 are short answer (SA) type questions, carrying 03 marks each.
6. In Section D, Questions no. 32-35 are long answer (LA) type questions, carrying 05 marks each.
7. In Section E, Questions no. 36-38 are case study based questions carrying 4 marks each.

## SECTION - A

Q.1 Function $f : N \to N$ defined by $f(x) = x^2 + x + 1$ is:

(a) one-one and onto (b) one-one but not onto

(c) onto but not one-one (d) neither one-one nor onto

Q.2 Given that A is square matrix of order 3×3 with $|A| = -5$ the $|AdjA|$ is:

(a) 5 (b) 25 (c) 125 (d) None

Q.3 If $y = \text{Log}\left(\sqrt{\text{Sec}\sqrt{x}}\right)$ then value of $\frac{dy}{dx}$ at $x = \frac{\pi^2}{16}$ is:

(a) $1/\pi$ (b) $\pi$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

Q.4 $\int \frac{x-3}{(x-1)^3} e^x dx$ is equal to:

(a) $\frac{2e^x}{(x-1)^3}$ (b) $\frac{-2e^x}{(x-1)^2}$ (c) $\frac{e^x}{(x-1)^2} + C$ (d) $\frac{e^x}{(x-1)} + C$

Q.5 The function $f(x) = 2x^3 - 3x^2 - 12x + 4$ has

(a) two points of local minima (b) two points of local maxima

(c) One maxima and one minima (d) No maxima no minima

Q.6 The side of an equilateral triangle is increasing at rate of 2cm /sec. The rate at which area increases when side is 10cm is:

(a) $10\ cm^2/5$ (b) $\sqrt{3}cm^2/s$ (c) $10\sqrt{3}\ cm^2/s$ (d) $\frac{10}{3}cm^2/5$

Q.7 The function $f(x) = ||1 - x + |x||$, $x \in R$ is

(a) Discontinuous at x=1 only (b) Discontinuous at x=0 only

(c) Discontinuous at x=0,1 (d) Continuous everywhere

Q.8 Value of x for which:

$A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix}$ is a Skew Symmetric matrix?

(a) 0 (b) 2 (c) –2 (d) –3

Q.9 If $y = Ae^{5x} + Be^{-5x}$ then $\frac{d^2y}{dx^2}$ is equal to

(a) 25y (b) 5y (c) -25y (d) 15y

Q.10 If $\int \frac{1}{(3+x)^2+1} dx = a\tan^{-1}(3+x) + C$ then value of a is:

(a) $\frac{1}{3}$ (b) 3 (c) 1 (d) -1

Q.11 If $\begin{vmatrix} 1 & 3 & 1 \\ k & 0 & 1 \\ 0 & 0 & 1 \end{vmatrix} = \pm 6$ then value of k is:

(a) 2 (b) – 2 (c) $\pm 2$ (d) None

Q.12 Derivative of Sin x with respect to log x is:

(a) $\frac{x}{Cos\,x}$ (b) $\frac{Cos\,x}{x}$ (c) $x\cos x$ (d) $x^2 Cos\,x$

Q.13 $\int \frac{\sqrt{\tan x}}{Sinx\ Cosx} dx$ is equal to :

(a) $2\sqrt{\cot x} + C$ (b) $\frac{\sqrt{\tan x}}{2} + c$ (c) $2\sqrt{\tan x} + c$ (d) None of these

Q.14 The value of :

$Cos^{-1}\left(\frac{1}{2}\right) + 3\ Sin^{-1}\left(\frac{1}{2}\right)$ is equal to:

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$ (c) $\frac{2\pi}{3}$ (d) $\frac{5\pi}{6}$

Q.15 The absolute maximum value of function $f(x) = x^3 - 3x + 2$ in $[0,2]$ is

(a) 0 (b) 2 (c) 4 (d) 5

Q.16 The function $y = x^2e^{-x}$ is decreasing in the interval.

(a) $(0, 2)$ (b) $(2, \infty)$ (c) $(-\infty, 0)$ (d) $(-\infty, 0) \cup (2, \infty)$

Q.17 If $\begin{vmatrix} x+3 & -2 \\ -3x & 2x \end{vmatrix} = 8, x \in N$ then value of x is:

(a) 1 (b) 5 (c) 2 (d) 4

Q.18 If $x = a\sec\theta, y = b\tan\theta$ the $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{6}$ is:

(a) $\frac{-3\sqrt{3}b}{a^2}$ (b) $\frac{-2\sqrt{3}b}{a}$ (c) $\frac{-3\sqrt{3}b}{a}$ (d) $\frac{-6}{3\sqrt{3}a^2}$

**Assertion- Reason Questions:**

Two statements are given, one is Assertion (A) and other is Reason (R). Select the correct answer.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

Q.19 Assertion (A): The function $f(x) = x + \frac{1}{x}$ is strictly increasing in the interval $(-1, 1) - \{0\}$

Reason (R): $f'(x) < 0$ for given interval.

Q.20 Assertion (A) : $f(x) = [x]$ is not differentiable at $x = 2$.

Reason (R) : $f(x) = [x]$ is not continuous at $x = 2$.

## SECTION – B

Q.21 If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ find k so that $A^2 = 5A + KI$, where I is identity matrix.

OR

If $x^y = e^{x-y}$ prove that $\frac{dy}{dx} = \frac{Logx}{(1+Logx)^2}$

Q.22 Find k so that given function.

$$f(x)\begin{cases}\frac{1-\cos 4x}{8x^2} & , x \neq 0 \\ k & , x = 0\end{cases}$$ is continuous at x=0.

OR

Evaluate $\int \frac{(x+1)(x+\log 2)^2}{x} dx$

Q.23 Sand is pouring from a pipe at the rate of $12cm^3/s$. The falling sand form a cone on the ground in such a way that the height of cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 4cm?

Q.24 Solve : $\int \frac{dx}{5-8x-x^2}$ → $\frac{1}{\sqrt{21}} \tan^{-1} \frac{x+4}{\sqrt{21}} + C$

Q.25 If $y = \sqrt{\sin x + y}$ the prove that $\frac{dy}{dx} = \frac{\cos x}{2y-1}$

## SECTION – C

Q.26 Differentiate $(\sin x)^{\log x} + (\sec x)^{\cot x}$

Q.27 Prove that:

$$\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right) = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x, \frac{-1}{\sqrt{2}} \leq x \leq 1$$

Q.28 If $A = \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$, I is identity matrix of order 2.

Show that

$$I+A=(I-A)\begin{bmatrix} Cos\,\alpha & -Sin\,\alpha \\ Sin\,\alpha & Cos\,\alpha \end{bmatrix}$$

Q.29 Find the intervals in which $f(x) = \sin x + \cos x$, for $0 \leq x \leq 2\pi$, is strictly increasing and strictly decreasing.

OR

Find the points of local maxima and local minima of $f(x) = -(x-1)^3(x+2)^3$.

Q.30 If $x = a\left(\text{Cost} + \log\tan\frac{t}{2}\right)$

$y = a\sin t$ find $\frac{d^2y}{dx^2}$

OR

Solve : $\int \frac{x^2+1}{(x-1)^2(x+2)}\,dx$

Q.31 Integrate $\int e^{-4x} \sin 3x\,dx$

SECTION – D

Q.32 If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$ find $A^{-1}$

using $A^{-1}$ solve the system of equations:

$2x - 3y + 5z = 11$

$3x + 2y - 4z = -5$

$x + y - 2z = -3$

Q.33 Integrate : (i) $\int \frac{dx}{1+\tan x}$ (ii) $\int \frac{1}{\sin(x+a)\sin(x+b)}$

Q.34 Consider $f : R_+ \to [-5, \infty]$ given by $f(x) = 9x^2 + 6x - 5$

Show that f is invertible with

$f^{-1}(y) = \frac{\sqrt{y+6}-1}{3}$

OR

A relation R on set A = {-4, -3, -2, -1, 0, 1, 2, 3, 4} be defined as

R {(x,y) : x+y is an integer divisible by 2}. Show that R is an equivalence relation. Also, find equivalence class [2].

Q.35 Express the following matrix as sum of symmetric and skew symmetric matrix and verify your result.

$$\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$$

OR

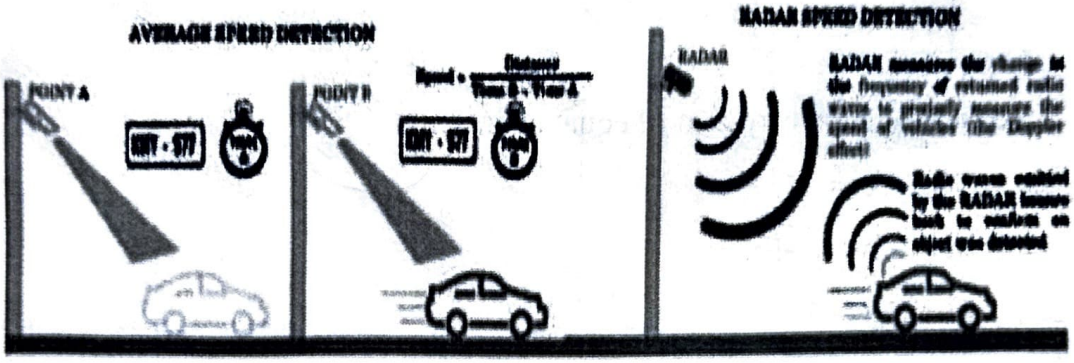
If $y = 3\cos(\log x) + 4\sin(\log x)$ Show that

$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

## SECTION – E

### CASE STUDY - 1

Q.36 The traffic police has installed Over Speed Violation Detection (OSVD) system at various locations in a city. These cameras can capture a speeding vehicle from a distance of 300 m and even function in the dark.




A camera is installed on a pole at the height of 5 m. It detects a car travelling away from the pole at the speed of 20 m/s. At any point, x m away from the base of the pole, the angle of elevation of the speed camera from the car C is θ . On the basis of the above information, answer the following questions:

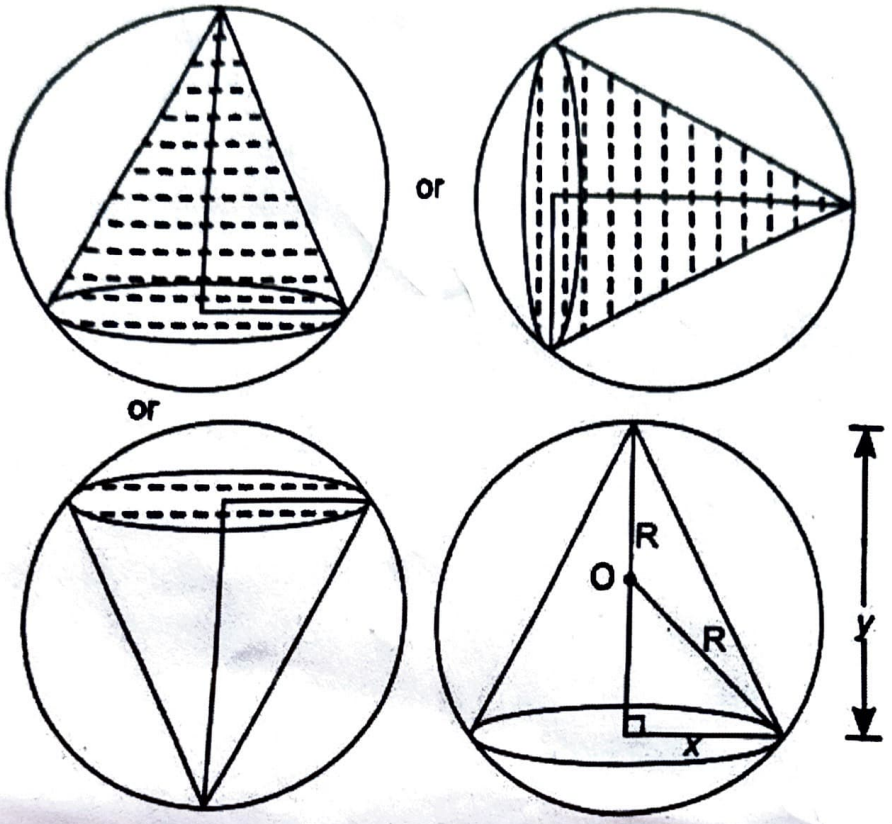
1.Express θ in terms of height of the camera installed on the pole and x. (1)

2. Find the rate of change of angle of elevation with respect to time at an instant when the car is 50 m away from the pole. (1)

3.If the rate of change of angle of elevation with respect to time of another car at a distance of 50 m from the base of the pole is $\frac{3}{101}$ rad/s, then find the speed of car (2)

### Case Study – 2

Q.37 A company dealing in gift items decided to make three-dimensional gift items and a core team was given responsibility for the same. After discussions core team suggested inserting the right circular cone in a sphere touching the surface of a sphere and at the same time gets illuminated when taking rounds. They presented the different dimensions, also keeping the cost factor into account. The gift item looks like as shown.




(i) If a cone of base radius x and height y is inscribed in sphere of radius R then establish relation between x, y and R. (1)

(ii) What is the volume of a cone in terms of y alone? (1)

(iii) What should be the length of altitude of a cone so that the volume of inscribed cone is maximum? Also find maximum volume of cone. (2)

**Case Study – 3**

Q.38 The relation between the height of the plant (y in cm) with respect to exposure to sunlight is governed by the following equation $y = 4x - \frac{1}{2}x^2$, where x is the number of days exposed to sunlight.

Answer the following based on above information.

(i) Find rate of growth of plants with respect to sunlight. (1)

(ii) What are the number of days it will take for the plant to grow to the maximum height ? (1)

(iii) What is the maximum height of plant ? (2)