Khushi

SECOND TERMINAL EXAMINATION (2022-23)

CLASS - XII

SUBJECT : STANDARD MATHEMATICS

Time : 3 hrs **Max. Marks : 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory.
2. Section A has 18 MCO's and 02 Assertion-Reason based questions of **1 mark** each.
3. Section B has 5 Very Short Answer (VSA)-type questions of **2 marks** each.
4. Section C has 6 Short Answer (SA)-type questions of **3 marks** each.
5. Section D has 4 Long Answer (LA)-type questions of **5 marks** each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment **(4 marks each)** with sub parts.

SECTION A

(Multiple Choice Questions)

Each question carries 1 mark

Q1. The maximum number of equivalence relations on the set $A = \{1,2,3\}$ are

a) 1 b) 2 c) 3 d) 5

Q2. Let $f: [2, \infty) \to R$ be a function defined by $f(x) = x^2 - 4x + 5$, then the range of f is

a) R b) $[1, \infty)$ c) $[4, \infty)$ d) $[5, \infty)$

Q3. The function $f : \{1, 2, 3 \ldots\ldots\} \to \{1, 4, 9 \ldots\ldots\}$ defined by $f(x) = x^2$ is

a) bijective
b) injective but surjective
c) surjective but not injective
d) neither surjective nor injective

Q4. The value of $\sin^{-1}\left[\cos\left(\frac{33\pi}{5}\right)\right]$ is

a) $\frac{3\pi}{5}$ b) $-\frac{7\pi}{5}$ c) $\frac{\pi}{10}$ d) $-\frac{\pi}{10}$

Q5. If A and B are invertible matrices, then which of the following is not correct?

a) $Adj(A) = |A|\,A^{-1}$
b) $Det\,(A^{-1}) = [Det\,(A)]^{-1}$
c) $(AB)^{-1} = B^{-1}A^{-1}$
d) $(A+B)^{-1} = B^{-1} + A^{-1}$

Q6. The maximum value of $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1+\cos\theta & 1 & 1 \end{vmatrix}$ is (θ is a real number)

a) $\frac{1}{2}$ b) $\frac{\sqrt{3}}{2}$ c) $\sqrt{2}$ d) $\frac{2\sqrt{3}}{4}$

Q7. If $f(x) = \begin{cases} mx+1 \text{ if } x \le \frac{\pi}{2} \\ \sin x + n \text{ if } x > \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$ then

a) $m = 1, n = 0$ b) $m = \frac{n\pi}{2} + 1$ c) $n = \frac{m\pi}{2}$ d) $m = n = \frac{\pi}{2}$

Q8. A box contains 3 orange balls, 3 green balls and 2 blue balls. Three balls are drawn at random from the box without replacement. The probability of drawing 2 green balls and one blue ball is

a) $\frac{3}{28}$ b) $\frac{2}{21}$ c) $\frac{1}{28}$ d) $\frac{167}{168}$

Q9. If $y = \sqrt{\sin x + y}$, then $\frac{dy}{dx}$ is equal to

a) $\frac{\cos x}{2y - 1}$ b) $\frac{\cos x}{1 - 2y}$ c) $\frac{\text{six } x}{1 - 2y}$ d) $\frac{\text{six } x}{2y - 1}$

Q10. The maximum value of $\left(\frac{1}{x}\right)^x$ is

a) e b) e^e c) $\frac{1}{e^e}$ d) $\left(\frac{1}{e}\right)^{\frac{1}{e}}$

Q11. If $P(A) = \frac{3}{10}$, $P(B) = \frac{2}{5}$ and $P(A \cup B) = \frac{3}{5}$, then $P(B/A) + P(A/B)$ equals

a) $\frac{1}{4}$ b) $\frac{1}{3}$ c) $\frac{5}{12}$ d) $\frac{7}{12}$

Q12. $\int_2^8 \frac{\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}}$ dx is equal to

a) 3 b) 6 c) 1 d) 10

Q13. $\int e^x \left(\frac{1-x}{1+x^2}\right)^2$ dx is equal to

a) $\frac{e^x}{1+ x^2} + c$ b) $-\frac{e^x}{1+ x^2} + c$

c) $\frac{e^x}{(1+ x^2)^2} + c$ d) $-\frac{e^x}{(1+ x^2)^2} + c$

H

Q14. If $\int \frac{x^3\,dx}{\sqrt{1+x^2}} = a(1+x^2)^{\frac{3}{2}} + b\sqrt{1+x^2} + c$, then

a) $a = \frac{1}{3}$, $b = 1$

b) $a = -\frac{1}{3}$, $b = 1$

c) $a = -\frac{1}{3}$, $b = -1$

d) $a = \frac{1}{3}$, $b = -1$

Q15. $\int \frac{dx}{e^x + e^{-x}}$ is equal to

a) $\tan^{-1}(e^x) + c$

b) $\tan^{-1}(e^{-x}) + c$

c) $\log(e^x - e^{-x}) + c$

d) $\log(e^x + e^{-x}) + c$

Q16. $\int \frac{\cos 2x \quad dx}{(\sin x + \cos x)^2}$ is equal to

a) $\frac{-1}{\sin x + \cos x} + c$

b) $\log|\sin x + \cos x| + c$

c) $\log|\sin x - \cos x| + c$

d) $\frac{1}{(\sin x + \cos x)^2} + c$

Q17. The integrating factor of the differential equation is $\frac{dy}{dx} + y = \frac{1+y}{x}$

a) $\frac{x}{e^x}$

b) $\frac{e^x}{x}$

c) $x\,e^x$

d) e^x

Q18. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three vectors such that $\vec{a} + \vec{b} + \vec{c} = 0$, and $|\vec{a}| = 2$, $|\vec{b}| = 3$ and $|\vec{c}| = 5$, then the value of $\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}$ is

a) 0

b) 1

c) -19

d) 38

$2\cdot3 + 3\cdot5 + 5\cdot2$

$= 6 + 15 + 10$

25

16

31

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R).

Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

Q19. A : Direction cosines of a line are the sines of the angles made by the line with the negative directions of the coordinate axes.

R: The acute angle between the lines $x - 2 = 0$ & $\sqrt{3}x - y - 2$ is $30°$

Q20. A: $f(x) = [x]$ is not differentiable at $x = 2$.
R: $f(x) = [x]$ is not continuous at $x = 2$.

SECTION B

This section comprises of very short answer type-questions (VSA) of 2 marks each

Q21. Show that the function 'f' defined on set $A = R - \left(\frac{2}{3}\right)$ defined as $f(x) = \dfrac{4x+3}{6x-4}$ is a bijection.

OR

Show that the relation R defined on R as $R = \{(a, b): a \leq b\}$, is reflexive and transitive but not symmetric.

Q22. Find the value of $\left(2\ \tan^{-1}\frac{1}{4}\right) + \cos\left(\tan^{-1} 2\sqrt{2}\right)$

Q23. Show that $f(x) = 2x + \cot^{-1} x + \log\left(\sqrt{1+x^2} - x\right)$ is increasing in R.

Q24. Find $\displaystyle\int \frac{2^{x+1} - 5^{x-1}}{10^{x}}\, dx$

Q25. A speaks truth in 80% cases and B speaks truth in 90% cases. In what percentage of cases are they likely to agree with each other in stating the same fact?

OR

A committee of 4 students is selected at random from a group consisting 8 boys and 4 girls. Given that there is atleast one girl on the committee, calculate the probability that there are exactly 2 girls on the committee.

SECTION C

(This section comprises of short answer type questions (SA) of 3 marks each)

Q26. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find k so that $A^2 = 5A + KI$.

Q27. If $\log(x^2 + y^2) = 2\tan^{-1}\left(\frac{y}{x}\right)$, show that $\frac{dy}{dx} = \frac{x+y}{x-y}$.

Q28. The side of an equilateral triangle is increasing at the rate of 2 cm/sec. At what rate is its area increasing when the side of the triangle is 20 cm?

OR

Find the intervals in which $f(x) = \text{Sin } 3x - \text{Cos } 3x$, $0 < x < \pi$, is strictly increasing or strictly decreasing.

Q29. Find $\int \frac{2\cos x}{(1-Sin\, x)(1+\sin^2 x)}dx$

Q30. Find the equation of the line which intersects the lines

$\frac{x+2}{1} = \frac{y-3}{2} = \frac{z+1}{4}$ & $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and passes through the point (1,1,1).

OR

Find the shortest distance between the lines

$$\vec{r} = (\hat{\imath} + 2\hat{\jmath} + \hat{k}) + \lambda(\hat{\imath} - \hat{\jmath} + \hat{k})$$

$$\& \vec{r} = 2\hat{\imath} - \hat{\jmath} - \hat{k} + \mu(2\hat{\imath} + \hat{\jmath} + 2\hat{k})$$

Q31. Solve the differential equation:

$x\,dy - y\,dx = \sqrt{x^2 + y^2}\,dx$, given that $y = 0$ when $x = 1$.

OR

Solve the differential equation:

$dy = \cos x\,(2 - \operatorname{cosec} x)\,dx$ given that $y = 2$ when $x = \frac{\pi}{2}$.

SECTION D

(This section comprises of long answer-type questions (LA) of 5 marks each)

Q32. If $A = \begin{bmatrix} 1 & 3 & 4 \\ 2 & 1 & 2 \\ 5 & 1 & 1 \end{bmatrix}$, find A-1. Hence solve the system of equations

$x + 3y + 4z = 8,\ 2x + y + 2z = 5\ \&\ 5x + y + z = 7.$

Q33. Draw a rough sketch of the region $\{(x,y): y^2 \leq 6ax \text{ and } x^2 + y^2 \leq 16a^2\}$. Also find the area of the region sketched using method of integration.

Q34. Find the foot of perpendicular from P (1,2, -3) to the line $\frac{x+1}{2} = \frac{y-3}{-2} = \frac{z}{-1}$. Also, find the image of P in the given line.

$\cos^2 x + \sin^2 x = 1$

$\sin^2 x = 1 - \cos^2 x$

$1 + \cos^2 x$

$(\cos x)^2$

$2\cos x \cdot (-\sin x) = dt$

$(\sin x)^2$

$2\sin x$

Surface area of cylinder

Q35. Solve the Linear Programming Problem graphically :-

Maximise $Z = 5x + 3y$
Subject to

$3x + 5y \leq 15$
$5x + 2y \leq 10$
$x \leq 0, y \leq 0$ $\quad$ $x \geq 0, y \geq 0$

OR

Determine graphically the minimum value of the objective function.

$Z = 50x + 20y$ $\quad$ $z = -50x + 20y$
Subject to constraints

$2x - y \geq -5$
$3x = y \geq 3$ $\quad$ $x + y \geq 3$
$2x - 3y \leq 12$
$x \geq 0, y \geq 0$

SECTION E

Q36. **CASE STUDY 1:** A tin can manufacturer designs a cylinderical tin canfor a company making sanitizer and disinfector. The tin can is made to hold 3 litres of sanitizer or disinfector. Based on the above information, answer the following questions:

i. If r cm be the radius and h cm be the height of the cylinderical tin can, then express surface area as a function of r. (1)

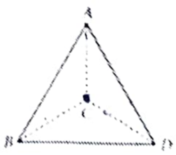
ii. Find the radius that will minimize the cost of the material to manufacture the tin can? (2)

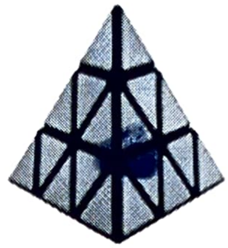
iii. Find the height that minimizes the cost of the material to manufacture the tin can? (1)

$\frac{1}{3}\pi r^2 h = 3$

Q37. **CASE STUDY 2:**

A building is to be constructed in the form of a triangular pyramid, ABCD as shown in the figure





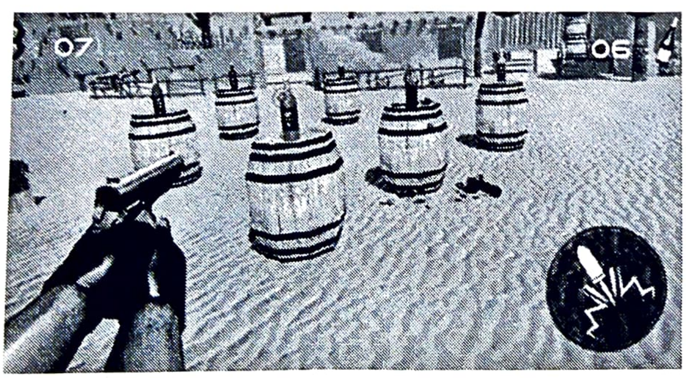
Let its angular points be A (0, 1, 2), B (3, 0, 1) C (4, 3, 6) & D (2, 3, 2) & G be the point of intersection of the medians of ΔBCD. Based on the above information, answer the following questions:

i. Find the coordinates of the point G? (1)

ii. What is the sum of the lengths of $\overrightarrow{AB}$ & $\overrightarrow{AC}$ (1)

iii. Find the area of ΔABC in square units. (2)

Q38. **CASE STUDY 3:**



Varun, Abhay and Rohit were doing shooting. Varun hits the target 4 times in 5 shots, Abhay hits the target 3 times in 4 shots and Rohit hits the target 2 times in 3 shots. Now on the basis of this information, answer the following questions:

i. Find the probability that any two among Varun, Abhay & Rohit may hit the target? (2)

ii. What is the probability that none of them will hit the target? (2)