Sahira

Cheating Replaces Learning

Term - I

Time Allowed: 3 Hours
Maximum Marks: 80

BVP/XII/Mathematics/2024-25

General Instructions: -

1. This Question Paper contains five sections - A, B, C, D and E. Each section is compulsory.
2. Section A has 18 MCQs and 2 Assertion –Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA) type questions of 2 marks each.
4. Section C has 6 Short Answer type questions of 3 marks each.
5. Section D has 4 Long Answer type questions of 5 marks each.
6. Section E has 3 source based /case based/passage based/integrated units of assessment 4 marks.

SECTION A (1 mark each)

Q1. Let L denote the set of all straight lines in a plane. Let a relation R be defined by $\alpha R \beta \Leftrightarrow \alpha \perp \beta$, $\alpha, \beta \in L$. Then R is

a. Reflexive
b. Symmetric
c. None of these
d. Transitive

Q2. A has 3 elements and the set B has 4 elements. Then the number of injective mapping that can be defined from A to B is

a. 144
b. 12
c. 24
d. 64

Q3. The value of $\tan(\sin^{-1} x)$ is

a) $\frac{x}{\sqrt{1+x^2}}$

b) $\frac{x}{\sqrt{1+x^2}}$

c) $\frac{\sqrt{1-x^2}}{x}$

d) $\frac{\sqrt{1+x^2}}{x}$

Q4.Domain of $\sin^{-1}(x^2 - 4)$ is

a. $[-\sqrt{5}, -\sqrt{5}]$
b. $[-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]$
c. $[-\sqrt{5}, -\sqrt{3}]$
d. $[\sqrt{3}, \sqrt{5}]$

Q5.If P is a 3 x 3 matrix such that P'=2P+1, where P' is the transpose of P , then

a. P= I
b. P= -I
c. P= 2I
d. P= -2I

Q6.Given that matrices A and B are of order 3 x n and m x 5 respectively, then the order of matrix C= 5A+3 B is

a. 3x5 and m=n
b. 3x5
c. 3x3
d. 5x5

Q7.Let A be a skew matrix of order 3. If $|A|$=x then $(2024)^x$ is equal to

a. 2024
b. 1/2024
c. 0
d. 1

Q8. If A.adjA=$\begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix}$, then the value of $|adjA|$ is

a. 12
b. 9
c. 3
d. 27

Q9. The function f(x)=$\frac{x-1}{x(x^2-1)}$ is discontinuous at

a) Exactly one point
b) Exactly two points
c) Exactly three points
d) No points

Q10.The value of k for which the function $f(x) = \begin{cases} x^2, & x \geq 0 \\ kx, & x < 0 \end{cases}$ is differentiable at x=0 is

a. 1
b. 2
c. Any real number
d. 0

Q11.The total revenue in rupees received from the sale of x units of a product is given by $R(x) = 3x^2 + 36x + 5$, the marginal revenue when x=15 is

a. 116
b. 96
c. 90
d. 126

Q12.$\int 2^{x+2}\,dx$ is equal to

a. 2^{x+2}+C
b. $2^{x+2}\,log2$+C
c. $\frac{2^{x+2}}{log2}$+C
d. $2.\frac{2^{x}}{log2}$+C

Q13.Antiderivative of $\frac{tanx-1}{tanx+1}$ with respect to x is

a. $sec^2(\frac{\pi}{4} - x)$+C
b. $-sec^2(\frac{\pi}{4} - x)$+C
c. $\log\left|sec(\frac{\pi}{4} - x)\right|$+C
d. $-\log\left|sec(\frac{\pi}{4} - x)\right|$+C

Q14.The area of curve y=sinx between 0 and π is

a. 2 sq. units
b. 4 sq. units
c. 12 sq. units
d. 14 sq. units

Q15.Which of the following is not a homogeneous function of x and y ?

a. x^2+xy
b. $y - 2x$
c. $\cos^2(\frac{y}{x}) + \frac{y}{x}$
d. sinx - cosy

Q16. What is the product of the order and degree of the differential equation $\frac{d^2y}{dx^2}\sin y+\left(\frac{dy}{dx}\right)^3\cos y=\sqrt{y}$.

a. 3
b. 2
c. 6
d. Not defined

Q17. In an LPP , if the objective function Z= ax+by has the same maximum value on two corner points of the feasible region , then the number of points of which Z_{max} occurs is

a. 0
b. 2
c. Finite
d. Infinite

Q18.The objective function Z=ax+by of an LPP has maximum value 42 at (4, 6) and minimum value 19 at (3, 2). Which of the following is true?

a) a=9, b=1
b) a=9, b=2
c) a=3, b=5
d) a=5, b=3

ASSERTION – REASON BASED QUESTIONS:

Directions: Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.,

(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion

(c) Assertion is correct, reason is incorrect

(d) Assertion is incorrect, reason is correct.

Q19. **Assertion (A):** The rate of change of area of a circle with respect to its radius r when r=6 cm is 12π cm^2.

Reason(R): The rate of change of area of a circle with respect to its radius r is $\frac{dA}{dR}$ where A is the area of the circle.

Q20. **Assertion (A):** Area enclosed by the circle $x^2 + y^2 = 36$ is 36 π sq. units.

Reason (R): Area enclosed by the circle $x^2 + y^2 = r^2$ is πr^2.

SECTION B (2 marks each)

Q21. Evaluate $\int_0^2 |x-1| dx$

OR

Evaluate: $\int_0^1 x^2 e^x dx$.

Q22. Solve the differential equation $\frac{dy}{dx}$=1+x+y+xy.

OR

Solve the differential equation $\frac{dy}{dx}$+2y tanx=sinx.

Q23. If siny=xcos(a+y), then $\frac{dx}{dy} = \frac{\cos a}{\cos^2(a+y)}$.

Q24. If y= $x^{1/x}$. then find $\frac{dy}{dx}$ at x=1.

Q25. Find the value of x for which y=$[x(x-2)]^2$ is an increasing function.

SECTION C (3 marks each)

Q26. The volume of a sphere is increasing at the rate of 3 cubic centimetre per second. Find the rate of increase in its surface area, when the radius is 2 cm.

Q27. A relation R is defined on a set of real numbers $\mathbb{R}$ as R= {(x,y): x.y is an irrational number}. Check whether R is Reflexive, symmetric and transitive or not.

OR

Consider f: $R_+ \rightarrow [4,\infty)$ given by f(x)=x^2+4. Check whether f is invertible or not?

Q28. Solve for x: $\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$.

OR

Solve $\cos(\tan^{-1} x) = \sin(\cot^{-1}\frac{3}{4})$.

Q29. Let $A=\begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$ $B=\begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}$, $C=\begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}$, Find a matrix D such that CD - AB=O

Q30. If $x=e^{\cos 2t}$ and $y=e^{\sin 2t}$, prove that $\frac{dy}{dx}= -\frac{y\log x}{x\log y}$.

OR

Differentiate $\sin^{-1}\left[\frac{2^{x+1}3^x}{1+36^x}\right]$ w.r.t.x.

Q31. Show that the points A (a, b+c), B(b,c+a), C(c,a+b) are collinear.

SECTION D (5 marks each)

Q32. Find the area of the region bounded by $4y=3x^2$ and the line $2y=3x+12$.

Q33. A tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth is 2m and volume is $8m^3$. If building of tank costs Rs. 70 per sq metres for the base and Rs. 45 per square metre for sides. What is the cost of least expensive tank?

OR

A wire of length 28m is cut into two pieces. One of the pieces is be made into a square and the other into a circle. What should be the length of the two pieces so that combined area of the square and the circle is minimum using differentiation?

Q34. An aeroplane can carry a maximum of 200 passengers. A profit of Rs 1000 is made on each executive class ticket and a profit of Rs 600 is made on each economy class ticket. The airline reserves at least 20 seats for executive class. However at least 4 times as many passengers prefer to travel by economy class than by the executive class. Determine how many tickets of each must be sold in order to maximize the profit for the airline. What is the maximum profit?

Q35. Evaluate $\int \frac{2x}{(x^2+1)(x^2+2)}$ dx.

OR

Evaluate $\int \frac{5x-2}{3x^2+2x+1}dx$

SECTION E (4 marks each)

This section comprises of 3 case- study/passage based questions of 4 marks each with sub parts.

Q36. Polio drops are delivered to 50K children in a district. The rate at which polio drops are given is directly proportional to the number of children who have not been administered the drops. By the end of 2nd week half the children have been given the polio drops. How many will have been given the drops by the end of 3rd week can be estimated using the solution to the differential equation $\frac{dy}{dx} = k(50 - y)$ where x denotes the number of weeks and y the number of children who have been given the drops.

i. (a) Find the solution of the differential equation $dy/dx = k(50 - y)$.
(b) Find the value of c in the particular solution given that $y(0) = 0$ and $k = 0.049$.

ii. Find the solution that may be used to find the number of children who have been given the polio drops?

Q37. The use of electric vehicles will curb air pollution in the long run. The use of electric vehicles is increasing every year and estimated number of electric vehicles in use at any time t is given by the function $V(t) = t^3 - 3t^2 + 3t - 100$ Where t represents time and t = 1, 2, 3, --------- corresponds to year 2021, 2022, 2023 ------- respectively.

Based on the above information answer the following:

i. Can the above function be used to estimate number of vehicles in the year 2020? Give reasons.

ii. Prove that the function V (t) is an increasing function.

Q38. The monthly incomes of two sister Reshma and Ritam are in the ratio 3:4 and their monthly expenditures are in the ratio 5:7. Each sister saves ₹15,000 per month.

i. Represent the given data in linear system of Equations and further represent the same in Matrix form.

ii. Find their monthly income and expenditure by Matrix Method.

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