

Half Yearly Examination (2023-2024)
Mathematics
Class - XI

Time: 3 hrs

M.M. 80

Section - A [One mark]

1. Let $A = \{1, 2\}$ and $B = \{3, 4\}$, how many subsets will $A \times B$ have

- (a) 8 (b) ~~16~~ (c) 32 (d) none of these

2. Let $A = \{9, 10, 11, 12, 13\}$ and $f: A \rightarrow N$ be defined by $f(x) =$ the highest prime factor of x . What is range of f

- (a) $\{3, 5\}$ (b) $\{3, 5, 11\}$ (c) $\{5, 11, 13\}$ (d) ~~$\{3, 5, 11, 13\}$~~

3. Let N be the number of subsets of the set formed with the letters of the word 'BLOSSOM'; then:

- (a) $N > 32$ (b) $N = 64$ (c) $N = 128$ (d) ~~$N = 32$~~

$\{B, L, O, S, M\}$

4. For any two sets A and B , the value of $A \cup (A \cap B)$ is:

- (a) ϕ (b) ~~A~~ (c) B (d) none of these

5. The greatest value of $\sin x \cos x$ is:

- (a) ~~1~~ (b) 2 (c) $\sqrt{2}$ (d) ~~$\frac{1}{2}$~~

6. The value of $\sin 240^\circ$ is:

- (a) $-\frac{1}{2}$ (b) $\frac{1}{2}$ (c) $\frac{\sqrt{3}}{2}$ (d) ~~$-\frac{\sqrt{3}}{2}$~~

7. If $\alpha + \beta = \frac{\pi}{4}$, then the value of $(1 + \tan \alpha)(1 + \tan \beta)$ is:

- (a) 1 (b) 2 (c) -2 (d) not defined

8. $1 + i^2 + i^4 + i^6 + \dots + i^{2n}$ is:

- (a) positive (b) ~~negative~~ (c) 0 (d) Can't be evaluated

$1 + (-1) + 1 + (-1) + 1 + (-1) + \dots + (-1)$

9. If $\left(\frac{1+i}{1-i}\right)^x = 1$, then for $n \in \mathbb{N}$

- (a) $x = 2n + 1$ (b) ~~$x = 4n$~~ (c) $x = 2n$ (d) $x = 4n + 1$

$\frac{\sqrt{3}}{2} \times \frac{1}{2}$
 $= \frac{\sqrt{3}}{4}$



10. If $-3x+17 < -13$, then:

- (a) $x \in (10, \infty)$ (b) $x \in [10, \infty)$ (c) $x \in (-\infty, 10]$ (d) $x \in [-10, 10]$

11. The marks obtained in first two tests are 90 and 80. Then the minimum marks that are required in the third test to have an average of at least 75 marks is:

- (a) 75 (b) 60 (c) 55 (d) 50

12. Total number of words formed by 2 vowels and 3 consonants taken from 4 vowels and 5 consonants is equal to:

- (a) 60 (b) 120 (c) 7200 (d) 720

13. If ${}^nC_{12} = {}^nC_8$, then n is equal to:

$\frac{20!}{12!8!} = \frac{20!}{12!8!}$

- (a) 20 (b) 12 (c) 6 (d) 30

$a = -1$
 $2 = -1$

14. Everybody in a room shakes hands with everybody else. The total number of handshakes is 66. The total number of persons in the room is:

$\frac{66 \times 2}{2}$
 132

- (a) 11 (b) 12 (c) 13 (d) 14

$a_n = -1(-1)^{n-1}$
 $-1/n$

15. The number of terms in the expansion of $(x+y)^n$ is:

- (a) $n+1$ (b) n^2 (c) $2n$ (d) n

16. The value at the 6th row and 3rd place in the Pascal's triangle is:

- (a) 5 (b) 10 (c) 15 (d) 20

17. The sequence $-1, 1, -1, 1, \dots$ can be expressed as: $-1(-1)^{n-1}$

- (a) $a_n = (-1)^{n+1}, n \in \mathbb{N}$ (b) $a_n = (-1)^n, n \in \mathbb{Z}$
(c) $a_n = (-1)^{n-1}, n \in \mathbb{N}$ (d) $a_n = (-1)^{2n}, n \in \mathbb{N}$

$\frac{24}{y} = \frac{24}{24}$
 $\frac{24}{y} = \frac{24}{24}$

18. If $x, 2y, 3z$ are in A.P. where distinct numbers x, y, z are in G.P. Then the common ratio of G.P. is:

$2 = 1 + r$
 $r = 2 - 1 = 1$

- (a) 3 (b) $\frac{1}{3}$ (c) 2 (d) $\frac{1}{2}$

$2 \times 3 = \frac{2}{2} \times 3$

$2 \times \frac{1}{2} = \frac{3 \times 1}{2 \times 2}$

$2y - x = 3z - 2y$

$2 \times \frac{1}{3} = \frac{2}{2} \times \frac{1}{3}$

$4y^2 = 3z \cdot x$

$4y = 3z + x$

ASSERTION - REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

- (a) Both A and R are true and R is the correct explanation of A.
- (b) Both A and R are true but R is not the correct explanation of A.
- (c) A is true but R is false.
- (d) A is false but R is true.

d 19. Assertion (A): The number of non zero integral solution of $|1-i|^x = 2^x$ is one.

Reason (R): $x = 0$ is solution of this equation.

$$(\sqrt{2})^x = 2^x$$

a 20. Assertion (A): Value of $\sin 15^\circ$ is $\frac{\sqrt{3}-1}{2\sqrt{2}}$

Reason (R): $\sin 15^\circ = \sin(45^\circ - 30^\circ)$ is solution of this equation.

Section - B [Two marks]

21. Find domain and range of $f: \mathbf{R} \rightarrow \mathbf{R}$ defined by $f(x) = [x]$ where $x \in \mathbf{R}$ assumes the value of greatest integer less than or equal to x

Or

Find domain and range of following real functions $f(x) = \sqrt{9-x^2}$

22. If $A = \{3, 5, 7, 9, 11\}$
 $B = \{7, 9, 11, 13\}$
 $C = \{11, 13, 15\}$

Find $(A \cap B) \cap (B \cup C)$

23. Prove:
$$\frac{\tan\left(\frac{\pi}{4} + x\right)}{\tan\left(\frac{\pi}{4} - x\right)} = \left(\frac{1 + \tan x}{1 - \tan x}\right)^2$$

Or

Prove that:
$$\cos\left(\frac{3\pi}{2} + x\right) \cos(2\pi + x) \left[\cot\left(\frac{3\pi}{2} - x\right) + \cot(2\pi + x) \right] = 1$$

24. Find modules of $\frac{1+i}{1-i} - \frac{1-i}{1+i}$ $\frac{1+i+1-i}{1+i-1+i} = \frac{2}{2i} = \frac{1}{i} = -i$

25. Sum of first three terms of a G.P. is $\frac{39}{10}$ and their product is 1. Find the common ratio.
 $\frac{1}{i} - \frac{2}{-i} = \frac{1}{i} + \frac{2}{i} = \frac{3}{i} = -3i$
 $\frac{1}{i} + \frac{2}{i} = \frac{3}{i}$
 $-i + i = 0$

Section - C [Three marks]

26. Prove that $\cos 4x = 1 - 8\sin^2 x \cos^2 x$

$\frac{1}{i} - \frac{2}{-i} = \frac{1}{i} + \frac{2}{i} = \frac{3}{i} = -3i$

Or

Prove: $\tan 4x = \frac{4 \tan x (1 - \tan^2 x)}{1 - 6 \tan^2 x + \tan^4 x}$

$\frac{5x+3x}{4} = \frac{8x}{4} = 2x$
 $\frac{5x-3x}{4} = \frac{2x}{4} = \frac{x}{2}$
 $\cos 5x - \cos 3x = -2 \sin 4x \sin x$

27. Prove that $\frac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x} = 2 \sin x$

28. If $a + bi = \frac{(x+i)^2}{2x^2+1}$ Prove that $a^2 + b^2 = \frac{(x^2+1)^2}{(2x^2+1)^2}$

29. In how many of the distinct permutations of letters in MISSISSIPPI do:
 a) vowels come together b) vowels do not come together

30. Using binomial theorem, prove that $6^n - 5n$ leaves remainder 1 when divided by 25.

31. Find the sum to n terms of the sequence: 8, 88, 888, 8888,

Or

Sum of two numbers is 6 times their geometric mean, show that the numbers are in the ratio $(3+2\sqrt{2}) : (3-2\sqrt{2})$

$2 \cos 6x \cos 9x$
 $-(\cos \frac{15x}{2} + \cos \frac{3x}{2})$

Section - D [Five marks]

32. Prove that: $\cos^2 x + \cos^2 \left(x + \frac{\pi}{3}\right) + \cos^2 \left(x - \frac{\pi}{3}\right) = \frac{3}{2}$

$\cos^2 x + 2 \cos^2 x \cdot \cos \frac{\pi}{3}$

Or

$2 \cos 4x \cdot \cos x$
 $\cos \frac{5x}{2} + \cos \frac{3x}{2}$

Prove that: $\cos 2x \cos \frac{x}{2} - \cos 3x \cos \frac{9x}{2} = \sin 5x \sin \frac{5x}{2}$

$\cos^2 x + \cos^2 x = 1$
 $\frac{2 \cos^2 x - \cos^2 x + 1}{2}$

$\cos A \cos B = \cos \frac{2x+2x}{2} + \cos \frac{2x-x}{2} - \cos 6x + \cos 2x$
 $\cos 2x + \cos x - \cos 6x + \cos 2x$

$\frac{\cos 5x}{2} - \frac{\cos 15x}{2}$

$\rightarrow 2 \sin 10x + \sin 5x$

33. A manufacturer has 600 l of a 12% solution of acid. How many litres of a 30% acid solution must be added to it so that acid content of resulting mixture will be more than 15% but less than 18%?

34. What is number of ways of choosing 4 cards from a pack of 52 playing cards if:

- a) four cards are of same suit b) four cards are of four different suits
c) are face cards d) two are red cards and two are black cards
e) cards are of same colour

Or

If the letters of the word 'RANDOM' are arranged in all possible ways as listed in a dictionary, then what is the rank of the word 'RANDOM'?

35. Evaluate: $(\sqrt{3} + \sqrt{2})^6 - (\sqrt{3} - \sqrt{2})^6$

Section - E [Four marks]
Case Study Questions

36. A person is not feeling well, so he goes to a doctor. The doctor on examination finds his temperature varies between $30^{\circ}C$ and $35^{\circ}C$.

If conversion formula is given by $C = \frac{5}{9}(F - 32)$ where C and F represents temperature in degree Celsius and degree Fahrenheit, then:

- (i) What is the temperature which is numerically same for both Fahrenheit and Celsius?
(ii) If the temperature is $95^{\circ}F$, what is temperature in Celsius?
(iii) What is range of temperature in degree Fahrenheit?

37. A committee of 6 members has to be formed from 8 boys and 5 girls. In how many ways can this be done if the committee consists of?

- (i) exactly 3 girls (ii) no girls (iii) at least 3 girls

38. A person has 2 parents, 4 grandparents, 8 great grandparents and so on. Find:

- a) Number of his ancestors in 10^{th} generation preceding his own.
b) Number of his ancestors during the ten generations preceding his own.

16 10 15 20
16 15 20 15 6