18 Sept, 2025

SGGS COLLEGIATE PUBLIC SCHOOL, SECTOR-26, CHANDIGARH

CLASS-XII

MATHEMATICS

FIRST TERM

Time allowed : 3 hours **Maximum Marks : 80**

General Instructions:

(1) Q.1. to Q.18. are of 01 mark each. Q.19. and Q.20. are Assertion-Reason type questions of 01 mark each.
(2) Q.21. to Q.25. are of 02 marks each.
(3) Q.26. to Q.31. are of 03 marks each.
(4) Q.32. to Q.35. are of 05 marks each.
(5) Q.36. to Q.38. are Case – Study/ Source Based Questions of 04 mark each.

SECTION-A

Q.1. Let $A=\{1,2,3\}$. Then number of equivalence relations containing (1,2) is
(a) 1 (b) 2 (c) 3 (d) 4

Q.2. $\tan^{-1}\sqrt{3}-\sec^{-1}(-2)$ is equal to
(a) π (b) $\frac{-\pi}{3}$ (c) $\frac{\pi}{3}$ (d) $\frac{2\pi}{3}$

Q.3. Given that A is square matrix of order 3×3 with $|A|=-5$ then value of $|AdjA|$ is
(a) 5 (b) 25 (c) 125 (d) None

Q.4. If $x=a\sec\theta$, $y=b\tan\theta$ then $\frac{d^2y}{dx^2}$ at $\theta=\frac{\pi}{6}$ is
(a) $\frac{-3\sqrt{3}b}{a^2}$ (b) $\frac{-2\sqrt{3}b}{a}$ (c) $\frac{-3\sqrt{3}b}{a}$ (d) $\frac{-b}{3\sqrt{3}a^2}$

Q.5. Function $f:N\to N$ defined by $f(x)=x^2+x+1$ is
(a) one-one and onto (b) one-one but not onto
(c) onto but not one-one (d) neither one-one nor onto

Q.6. $\sin^{-1}\left(\cos\frac{13\pi}{5}\right)$ is
(a) $\frac{-3\pi}{5}$ (b) $\frac{-\pi}{10}$ (c) $\frac{3\pi}{5}$ (d) $\frac{\pi}{10}$

Q.7. $\int\frac{x^9}{(4x^2+1)^6}dx$ is equal to
(a) $\frac{1}{5x}\left(4+\frac{1}{x^2}\right)^{-5}+c$ (b) $\frac{1}{5}\left(4+\frac{1}{x^2}\right)^{-5}+c$
(c) $\frac{1}{10x}\left(\frac{1}{x^2}+4\right)^{-5}+c$ (d) $\frac{1}{10}\left(\frac{1}{x^2}+4\right)^{-5}+c$

Q.8. A cylindrical tank of radius 10 m is being filled with wheat at the rate of 314 cubic metre per hour. Then the depth of the wheat is increasing at the rate of
(a) 1 m/h (b) 0.1 m/h (c) 1.1 m/h (d) 0.5 m/h

Q.9. $\int_{-1}^{2}[x]dx$ where $[x]$ denote the greatest integer $\leq x$ is

(a) 2 (b) 1 (c) 3 (d) 0

Q.10. The function $f(x)=2x^3-3x^2-12x+4$ has

(a) two points of local maxima (b) two points of local minima

(c) one maxima and one minima (d) no maxima no minima

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Q.11. The interval in which $y=x^2e^{-x}$ is increasing is

(a) $(-\infty,+\infty)$ (b) $(-2,0)$ (c) $(2,\infty)$ (d) $(0,2)$

Q.12. If $y=f\left(\frac{1}{x}\right)$ and $f'(x)=x^3$ then what is value of $\frac{dy}{dx}$ at $x=\frac{1}{2}$?

(a) $\frac{-1}{64}$ (b) $\frac{-1}{32}$ (c) -32 (d) -64

Q.13. If $y=\log\sqrt{\sec\sqrt{x}}$ then value of $\frac{dy}{dx}$ at $x=\frac{\pi^2}{16}$ is

(a) $\frac{1}{\pi}$ (b) π (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

Q.14. $\int\frac{x-3}{(x-1)^3}e^x dx$ is equal to

(a) $\frac{2e^x}{(x-1)^3}+c$ (b) $\frac{-2e^x}{(x-1)^2}+c$ (c) $\frac{e^x}{(x-1)}+c$ (d) $\frac{e^x}{(x-1)^2}+c$

Q.15. Domain of $\sin^{-1}\sqrt{x-1}$ is

(a) [-1,1] (b) $\left[0,\frac{1}{2}\right]$ (c) [0,1] (d) [0,2]

Q.16. Find k if area of triangle formed by points (-2,0), (0,4), (0,k) is 4 sq. units

(a) 0,4 (b) 0,8 (c) 0,-4 (d) 0,-8

Q.17. $\int\frac{x^3}{\sqrt{1+x^2}}dx=2a(1+x^2)^{\frac{3}{2}}-b\sqrt{1+x^2}+c$ then

(a) $a=\frac{1}{3},b=1$ (b) $a=\frac{-1}{6},b=1$ (c) $a=\frac{-1}{3},b=1$ (d) $a=\frac{1}{6},b=1$

Q.18. The absolute maximum value of the function $f(x)=4x-\frac{1}{2}x^2$ in $\left[-2,\frac{9}{2}\right]$ is

(a) 8 (b) 9 (c) 6 (d) 10

Q.19. and Q.20. are Assertion- Reason Type Questions:

Two statements Assertion (A) and Reason (R) are given. You are to choose correct option.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

Q.19. **Assertion (A):** If $x = t^2, y = t^3$ then $\frac{d^2y}{dx^2} = \frac{3}{2}$

Reason (R): $\frac{dy}{dx} = \frac{3t}{2}, \frac{d^2y}{dx^2} = \frac{3}{2}\frac{dt}{dx} = \frac{3}{4t}$

Q.20. **Assertion (A):** The system of equations $x+2y=4,\ 2x+4y=8$ is consistent with unique solutions.

Reason (R): For a square matrix A in a matrix equation $Ax = B$, if $|A| \neq 0$ then system of equations is consistent with unique solution.

SECTION-B

Q.21. Find principal value of $\sin^{-1}\left(\frac{-1}{2}\right) + \tan^{-1}(-1) + \sec^{-1}(2)$

Q.22. If $x^y = e^{x-y}$, prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$

OR

Evaluate: $\int x^2 \tan^{-1} x\, dx$

Q.23. If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$. Find K so that $A^2 + 2I = KA$

Q.24. Find k so that given function is continuous?

$$f(x) = \begin{cases} \frac{1-\cos 4x}{8x^2} & , x \neq 0 \\ k & , x = 0 \end{cases}$$

at $x = 0$

Q.25. Check the differentiability of $f(x) = [x]$ at $x = 1$.

SECTION-C

Q.26. Differentiate: $(\log x)^{\log x} + (\tan x)^{\cos\sqrt{x}}$

Q.27. Prove that

$$\tan^{-1}\left[\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right] = \frac{\pi}{4} + \frac{1}{2}\cos^{-1} x^2$$

OR

$$\int \frac{(x^2+1)dx}{(x^2+4)(x^2+7)}$$

Q.28. Show that the relation R in the set $A = \{x \in Z : 0 \leq x \leq 12\}$ is defined by $R = \{(a,b) : |a-b|$ is a multiple of $4\}$ is an equivalence relation.

Q.29. $\int \frac{5x-4}{\sqrt{4x^2-3x-2}}$

OR

$4x^2 - 3x + \frac{9}{4}$

$4x^2 - 3x - 2 + \frac{9}{4} - \frac{9}{4}$

$$f(x)=\begin{cases}\dfrac{\sin(a+1)x+\sin x}{x}, & x<0\\[2mm] c, & x=0\\[2mm] \dfrac{\sqrt{x+bx^2}-\sqrt{x}}{bx^{\frac{3}{2}}}, & x>0\end{cases}$$

is continuous at $x=0$. Find the values of a, b, c.

Q.30. If $y=e^{a\cos^{-1}x}$ then show that $(1-x^2)\dfrac{d^2y}{dx^2}-x\dfrac{dy}{dx}-a^2y=0$

Q.31. Find the intervals in which $f(x)=-(x+1)^3(x-3)^3$ is strictly increasing. and strictly decreasing.

SECTION-D

Q.32. Solve using matrix method:

$x-y+2z=7$

$3x+4y-5z=-5$

$2x-y+3z=12$

Q.33. Is the function $f(x)=\dfrac{x-1}{x+1}$ invertible in its domain? If so, find its inverse

OR

(i) $\int e^{2x}\cos 4x\,dx$

(ii) $\int \dfrac{dx}{1-\cot x}$ log|sinx|.

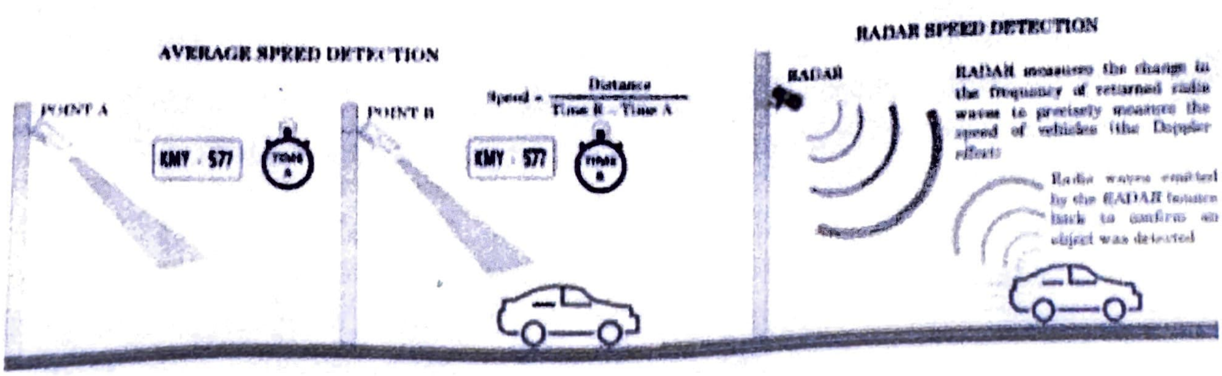
Q.34. Find local maximum and local minimum values of

$f(x)=\sin^4 x+\cos^4 x,\ 0<x<\dfrac{\pi}{2}$

Q.35. Sand is pouring from a pipe at the rate of 12 cm^3/ sec. The falling sand forms a cone on the ground in such a way that height of the cone is always one-sixth of the radius of the base. How fast is the height of sand cone increasing when the height is 4 cm? $\frac{1}{3}\pi r^2 h$ B = 2πrh + πr

CASE STUDY

Q.36. The traffic police has installed Over Speed Violation Detection (OSVD) system at various locations in a city. These cameras can capture a speeding vehicle from a distance of 300 m and even function in the dark.



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A camera is installed on a pole at the height of 5 m. It detects a car travelling away from the pole at the speed of 20 m/s. At any point, x m away from the base of the pole, the angle of elevation of the speed camera from the car C is θ.
On the basis of the above information, answer the following questions:

(i) Express θ in terms of height of the camera installed on the pole and x. 1

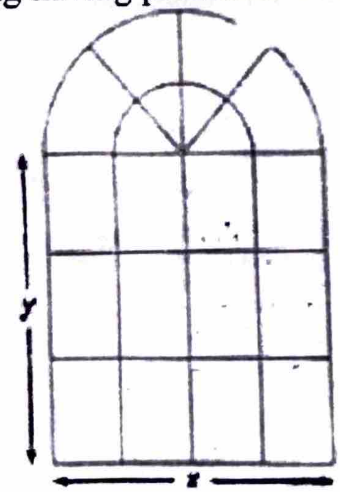
(ii) Find $\frac{d\theta}{dx}$ 1

(iii) (a) Find the rate of change of angle of elevation with respect to time at an instant when the car is 50 m away from the pole. 2

OR

(iii) (b) If the rate of change of angle of elevation with respect to time of another car at a distance of 50 m from the base of the pole is $\frac{3}{101}$ rad/s, then find the speed of the car. 2

Q.37. Rohan, a student of class XII, visited his uncles's flat with his father. He observe that the window of the house is in the form of a rectangle surmounted by a semicircular opening having perimeter 10 m as shown in the figure.



$2x+2y+\pi x = 10$

Based on the above information, give the answer of the following questions:

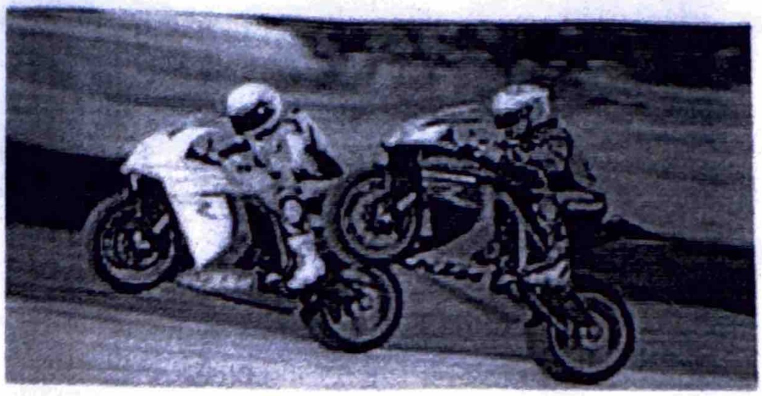
(i) If x and y represents the length and breadth of the rectangular region, then find the area (A) of the window in terms of x.

(ii) Rohan is interested in maximising the area of the whole window, for this to happen, find the value of x.

(iii) Find the maximum area of the window.

(iv) For maximum value of A, find the breadth of rectangular part of the window.

Q.38. An organization conducted bike ~~race~~ under 2 different categories-boys and girls. Totally there were 250 participants. Among all of them finally three from Category 1 and two from Category 2 were selected for the final race. Ravi forms two sets B and G with these participants for his college project.
Let $B = \{b_1, b_2, b_3\}$ $G = \{g_1, g_2\}$ where B represents the set of boys selected and G the set of girls who were selected for the final race.



Ravi decides to explore these sets for various types of relations and functions

(i) Ravi wishes to form all the relations possible from B to G. How many such relations are possible?

(ii) Ravi wants to know among those relations, how many functions can be formed from B to G?

(b_3, g_2)

(iii) Let $R: B \rightarrow G$ be defined by $R = \{(b_1, g_1), (b_2, g_2), \cancel{(b_3, g_3)}\}$, then R is _____.

(iv) Ravi wants to find the number of injective functions from B to G. How many numbers of injective functions are possible?